

Scheduling Access to Shared Space in Multi-Robot Systems

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Abstract. Through this study, we introduce the idea of applying scheduling techniques to allocate spatial resources that are shared among multiple robots moving in a static environment and having temporal constraints on the arrival time to destinations. To illustrate this idea, we present an exemplified algorithm that plans and assigns a motion path to each robot. The considered problem is particularly challenging because: (i) the robots share the same environment and thus the planner must take into account overlapping paths which cannot happen at the same time; (ii) there are time deadlines thus the planner must deal with temporal constraints; (iii) new requests arrive without a priori knowledge thus the planner must be able to add new paths online and adjust old plans; (iv) the robot motion is subject to noise thus the planner must be reactive to adapt to online changes. We showcase the functioning of the proposed algorithm through a set of agent-based simulations.

1 Introduction

Consider the example of a hospital where patients are transported to the required location within the hospital (e.g., a medical ward or an operating theater) and this transportation is performed by dedicated robots (e.g., the robots presented in [17]). Each patient may have a different medical situation which determines the urgency of the transportation, thus, a specific temporal deadline. Additionally, in most cases, the arrival of new patients cannot be predicted in advance but the system must deal with online requests. The robots that operate in this transportation system share a given environment, thus share the same limited resources. For instance, a lift or a corridor can be accessed by a limited number of robots at a time. Finally, robots may not have a deterministic arrival time, but their motion may be subject to delays (e.g., due to the avoidance of unexpected obstacles, such as humans, or due to a noisy robot motion). This example presents a very challenging problem which requires the robust online planning of paths for multiple robots with temporal and spatial constraints. In this study, we investigate this problem and propose an algorithm that deals with such types

of constraints. The proposed algorithm does not provide the complete solution that can be directly applied to this example but it is a significant step forward in such direction which tackles various challenging aspects.

The considered challenges can be ascribed as the core problems investigated in the two research areas of path planning and scheduling (which we review in Sect. 2). The former area studies solutions to plan the sequence of intermediate locations (configurations) that a robot has to visit (implement) for moving from a starting position to a final destination. Almost every mobile robot system has to deal with this aspect and, in fact, this research area has been very active since a few decades and several solutions have been proposed [3,7]. The latter research area, scheduling, studies how to plan the times of access to shared resources. Solutions in this area typically aim at problems of sharing computational power [2], while we are not aware of any work that considered the space as the resource that needs to be scheduled for a shared access under deadline constraints. Our work lays at the interface of these two areas; the main idea is to get inspiration from solutions in scheduling, and employ and adapt them for multi-robot path planning with arrival time deadlines. We illustrate this idea by proposing an exemplified time-space planner which we present in Sect. 3. Additional constraints that make the investigated problem more challenging, while closer to a real world application, are (i) stochasticity in robot motion and (ii) on-line requests for new paths. The proposed algorithm is reactive and thus able to modify online the planned paths in response to delays in the robot motion. Additionally, the algorithm evaluates online new requests and either rejects or accepts them. The rejection of a request means that there is no possible plan that would allow reaching the destination within the given deadline without cancelling already scheduled paths. In this case, possible solutions are either to cancel already planned paths, or to extend the temporal deadline. The current version of our algorithm does not make decisions of this type. Instead, when a request is accepted, the planner may modify, if necessary, other planned paths which may include the preemption (i.e., the pausing) of a robot that was moving to allow another robot with a higher priority to access to a spatial resource. We perform a set of agent-based simulations to verify the correctness and efficiency of our algorithm in Sect 4. Finally, we give final remarks on our study in Sect. 5.

2 Related Work

Path planning algorithms can be organised in two macro-categories: deterministic algorithms and stochastic algorithms. Deterministic algorithms are preferred when agents have few degrees of freedom that determine a limited number of possible configurations; here, the algorithm can exploit a tractable solution space and provide provable bounds on the solution quality. Instead, stochastic algorithms are preferred when the solution space to explore is extremely large and stochastic exploration is the only resort to speed up the planning time. In this work, we plan paths over a small graph that allows us to implement a deterministic algorithm. Among the most known deterministic algorithms for planning

are Dijkstra’s and A^* ; and several variants and extensions of these algorithms have been proposed (e.g., D^* , or the jump point search) [3,7]. Path planning in multi-robot systems has been often tackled as an optimization problem focused on finding the shortest collision-free path [1,15,11,16]. Some of these works [1,15] included priorities to give precedence to some robots in case of conflicting access to space. Differently from other works in multi-robot motion planning, our study takes into account a specific temporal deadline for each robot (thus, for each path) and hence the planner has to schedule movements both in space and in time.

Traditionally, real-time **scheduling** algorithms have dealt with deterministic execution times such as the worst case execution time [2]. While, stochastic execution times of tasks have been considered only in a limited number of studies. Some of them modified existing deterministic algorithms to support variable execution times, e.g., [10]. Others have implemented heuristic approaches to schedule tasks with stochastic execution times on multi-processors [9]. All these works, similarly to ours, have in common the use of probability distributions to characterize the stochastic execution time of the tasks. In fact, in our study, tasks are not pieces of code to execute (as in traditional scheduling), but tasks are robot motions which are characterized by stochastic execution times. Accounting for time constraints, such as deadlines, in multi-robot systems has been studied only in a limited number of works, e.g., [5,6].

Other works tackled the problem of **allocating resources in multiagent systems** through various methods such as continuous-time DEC-MDP and DEC-POMDP—decentralised (partially observable) markov decision processes—e.g. [18], or distributed negotiation protocols, e.g. [8], or metaheuristic approaches, e.g., in vehicle routing [13]. Differently from ours, this class of studies aim to minimize the costs of resource allocation and therefore solve an optimization problem. Instead, we assume specific deadlines for each task and the minimization of execution time is not required. Solving a minimization problem adds a level of complexity that is unnecessary for our problem; for example, tasks with very far deadlines can make a relaxed use of resources. This difference led us to tackle the problem in a different way, i.e., through a scheduling algorithm.

Another class of problems that presents strong similarities with our study is **railway scheduling** [14,4] where train journeys with given arrival times are scheduled. As a main difference, our study considers the unpredicted arrival of new tasks which requires to alter the previous schedule and to determine if accepting the new task is possible or not. Additionally, as presented in Sect. 3, we include the notion of link congestion which might allow us to consider the use of resource by entities external to the system under control (e.g. humans).

3 Time-space Planning Algorithm

The proposed algorithm plans access to a shared space complying with specific deadlines. We model the environment as a graph in which nodes represent locations and links represent connections between locations. We assume that a link

between two locations can be used by only one robot at a time (even if the other robots move in the opposite or in the same direction, e.g. a lift in a building). We assume a system composed of N identical robots. A robot may get a task T_i which is characterized by four attributes: the start time $a_{i,0}$, the start location, the destination, and the temporal deadline D_i . This information is not known a priori but becomes available to the planner only when a new task arrives. In our work we deal with stochastic execution times that can be characterized through its probability distribution.

When a new task arrives, the algorithm evaluates whether to accept or reject it through an *acceptance test*⁴. A task passes the acceptance test if (i) a feasible path connecting its start location to its destination is found (space planning) (ii) and if this path can be completed before the deadline expiration and without violating any deadline of previously-scheduled tasks (time planning). Our algorithm is reactive to unexpected delays in the robot motion which requires the adjustments of the generated time-space plan. A robot delay is treated as the arrival of a new task using as start location the location where the unexpected delay is reported and as a start time the time at which the robot arrived at that specific location. The acceptance test evaluation is composed of two phases: space planning and time planning. The goal of this study is to illustrate the utility of including a time planning phase—based on scheduling techniques—to allocate spatial resources in a multi-robot system. To highlight and measure the impact of the time planning phase in a clean way, we want to limit as much as possible the influence of arbitrary design choices in the space planning phase, e.g., possible stochastic components. To this end, in the proposed exemplified algorithm we rely on a complete and deterministic planning solution. This choice allows us to illustrate the advantages (and drawbacks) of the time planning phase through a set of experiments on a small size grid environment. In larger environments, this solution would not be viable and the space planning phase can be replaced by any stochastic planning algorithm [3,7].

Space Planning. In this phase, the algorithm finds and orders all feasible paths between the start location and the destination. These paths are ordered by their weight which is computed combining two measures: (i) the length of the path and (ii) the congestion of the links along the path. The path length is a deterministic measure that is defined as the sum of the lengths of all links in the path. The congestion of a link is a stochastic measure—characterised by a probability distribution—that represents the number of individuals (including robots and other entities: e.g. humans) that cross the link during a time unit. The congestion measure allows the system to operate on more realistic assumptions with which the robots are not isolated from other systems operating in the same environment. The algorithm uses the expected value of the congestion distribution. Since we are dealing with known environments, solid distributions for the links

⁴The term "acceptance test" or "schedulability test" is also used in traditional real-time systems to refer to the decision process of accepting or rejecting a task based on the ability of scheduling it under the given time constraints.

congestion can be obtained. These can be time-variant distributions, where the congestion over links varies over time. Our algorithm applies the expected value of the congestion according to the distribution valid during the time of planning. The algorithm combines these two measures (the path length and the path congestion) to compute a weight W_{π_j} for the j -th path π_j and then order the set of paths Π according to their weights. The weight W_{π_j} is computed as:

$$W_{\pi_j} = \alpha l_{\pi_j} + (1 - \alpha) c_{\pi_j}, \quad (1)$$

where l_{π_j} is the length measure and c_{π_j} is the congestion measure of the path π_j and α is a design parameter. The length l_{π_j} is a normalised value in the range $[0, 1]$ computed as $l_{\pi_j} = L_{\pi_j} / \text{Max}(L_{\pi})$ with $\text{Max}(L_{\pi})$ the longest path between any two locations in the environment (we do not consider loops). Similarly, the congestion c_{π_j} is normalised in the range $[0, 1]$ as $c_{\pi_j} = C_{\pi_j} / \text{Max}(C_{\pi})$ with $\text{Max}(C_{\pi})$ the maximum congestion over all paths in the environment. The computation of all feasible paths may be computationally expensive. We assume a static environment, therefore, the set of paths Π can be computed offline for all pairs of start locations and destinations. On the contrary, the congestion measure may possibly vary over time which requires the online computation of weights W_{π_j} and the online ordering of Π . Additionally, in case a link has a limited capacity (i.e., a limited number of robots can use a link at the same time), the congestion measure must be updated online each time a new path is planned. In this study, however, we assume that a link has a very limited capacity for robots: it can be used by only one robot at a time. This assumption allows us to simplify the algorithm by ordering Π offline and focusing on the scheduling algorithm to allocate one link at a time to each robot.

Time Planning. During this phase, the algorithm assesses the validity of the paths in Π sequentially following the ascending order of their weights. The selection of the parameter α determines the order of the paths. This parameter has no crucial effect on the performance of the algorithm since we consider an offline average of the link congestion and the goal is not to select the shortest path, but a path that respects the task deadline. The time planning phase algorithm is complete, thus, will evaluate all paths before rejecting a task, therefore, the choice of the parameter α may influence only the algorithm speed. A path is considered *valid* if it allows the robot to move between the start location and the destination before the task's deadline is expired and without violating any of the deadlines of already accepted tasks. When a valid path π_j (j is the order of the path in the set Π) is found, the task is accepted and the path π_j is assigned to the task without further checking of the remaining paths in Π .

Each link in the path π_j represents a spatial resource that could be shared among several tasks that may attempt to access it with time intersections. Hence, we need to schedule the time access to these links. We do so through the widely-used EDF (Earliest Deadline First) scheduling algorithm⁵. In traditional real-time

⁵EDF is a preemptive optimal scheduling algorithm for dynamic priorities. The tasks' priorities are updated during the execution of the tasks based on the current conditions.

systems, EDF uses the worst-case execution time to check the task's schedulability and to generate feasible schedules. For multi-robot systems, this is not trivial because of two challenges to overcome: First, the execution time of individual links (i.e., the time spent in crossing a link) is stochastic; Second, scheduling the access to a particular link influences the execution times of all further links used by the related tasks due to new preemptions which were not planned before accessing the link. Therefore, we propose the following approach to facilitate scheduling.

The execution time $\rho_i(h)$ of a link h by robot i on task T_i is a stochastic measure that we model using the normal probability distribution. The time is determined by two components: the speed and reliability of the robot resulting in the particular motion time e_i of the robot performing T_i , and the congestion on the link C_h , thus, $\rho_i(h) = e_i + C_h$. The distribution of $\rho_i(h)$ can be approximated by a normal distribution [12] according to the central limit theorem. Hence, $\rho_i(h) \sim \mathcal{N}(\mu, \sigma)$, $\forall i, h$. When links have different congestion, the normal distribution that models the execution time of each links has a different mean and standard deviation, nevertheless, it does not influence the computations of the algorithm. Following the statistical $3\text{-}\sigma$ rule of the normal distribution, the probability that the execution time of link is smaller than $\mu + 3\sigma$ is 0.99. Hence, considering the planning value $\rho_i(h) = \mu + 3\sigma$ allows the system to operate with the probability of having delays with respect of the planned time on link h minimized to $0.01 = 1 - 0.99$. Using this planning value is similar to planning with the worst-case execution time but considering 99% of the cases.

Evaluating the acceptance of path π_j for task T_i consists in checking if executing all links $h \in \pi_j$ (i.e. moving through them) complies with the deadline D_i and does not violate the deadlines of the already-scheduled tasks. To make this evaluation, the algorithm computes for task T_i its ready-to-run time $\theta_i(h)$ at each link h , which is the time at which the robot on task T_i is ready to move through link h . The expected ready-to-run time $\mathbb{E}(\theta_i(h))$ on link h is computed as:

$$\mathbb{E}(\theta_i(h)) = \mathbb{E}(a_i(h)) + \mathbb{E}(\gamma_i(h)) \quad (2)$$

where $\mathbb{E}(a_i(h))$ is the expected arrival time of robot i at link h and $\mathbb{E}(\gamma_i(h))$ is the expected preemption time of robot i before executing link h . The expected arrival time $\mathbb{E}(a_i(h))$ is computed as the sum of the planned times spent in crossing all the previous links $q \in \{1, \dots, h-1\}$ plus the times spent in preemption on the previous links:

$$\mathbb{E}(a_i(h)) = a_{i,0} + \sum_{q=1}^{h-1} (\rho_i(q) + \mathbb{E}(\gamma_i(q))) \quad (3)$$

where $a_{i,0}$ is the start time of task T_i and $\rho_i(q)$ is the estimated motion time (e.g., $\rho_i(q) = \mu + 3\sigma$) on link q .

The expected preemption time $\mathbb{E}(\gamma_i(h))$ at link h is calculated as a result of the dynamic priorities assigned by EDF to all the tasks requiring access to this particular link at the same time. These priorities are assigned based on the links' deadlines $D_i(h)$, $h \in \pi_j$. The deadline $D_i(h)$ of link h is computed as a fraction

of the total deadline D_i of task T_i :

$$\mathbb{E}(D_i(h)) = \mathbb{E}(a_i(h)) + \frac{l_h}{l_{\pi_j}} \times (D_i - a_{i,0}) \quad (4)$$

where l_h is the length of link h and l_{π_j} is the total length of path π_j . After computing the expected deadline of link h for all tasks that are attempting to access link h with time intersections, EDF assigns them dynamic priorities based on their computed deadlines (i.e., shorter deadline higher priority). After assigning the order in which tasks are allowed to execute link h , it becomes possible to compute the preemption times for the tasks at this link. For task T_i , this is given by:

$$\mathbb{E}(\gamma_i(h)) = \mathbb{E}(RE_s(h)) + \sum_{q=1}^r \rho_i(q) \quad (5)$$

where $RE_s(h)$ is the execution time left for task T_s —that was running when task T_i arrived at the link h —to finish executing link h . This execution time is zero when there is no task running over link h when task T_i arrives. Furthermore, r is the number of tasks with a higher priority than T_i . After the algorithm schedules the access to link h and computes the preemption times of all the tasks attempting to access this link, it updates the arrival times of these tasks at all the future links of their planned paths. This update may result in new time intersections which need to be scheduled. While updating the tasks' preemption times, the algorithm checks, at each link, whether there is any violation of the link deadline $D_i(h)$:

$$\mathbb{E}(\theta_i(h)) + \rho_i(h) \leq \mathbb{E}(D_i(h)) \quad (6)$$

If a violation appears on at least one of the links, the corresponding path is rejected. Otherwise, it is accepted and assigned to the robot.

Algorithm Complexity. The computations with the highest complexity are performed by the algorithm during the space planning phase in which the algorithm iterates over all possible paths between any two nodes. Since we are dealing with known static environments, all these computations are done *offline*, with complexity $O(mn^2)$. As mentioned above, we selected a complete (but expensive) space planning algorithm to remove any possible bias coming from the specific implementation and parameterization of a stochastic path planning algorithm. However, in case needed, the space planning phase could use a stochastic algorithm to save time and to generate a smaller set of paths.

Since robots (tasks) arrive online, the computations performed by the algorithm during the time planning phase are made *online*. For a given pair of start-end locations, the algorithm verifies the time constraints of the paths between these two locations. The worst case for the algorithm is when no path among these satisfies the time constraints (i.e., does not meet the corresponding task's deadline without violating previously planned paths). This is when the algorithm rejects the task after going over all these paths and checking Equation (6), which takes $(m - 1 + n - 1)!/(m - 1)!(n - 1)!$ time.

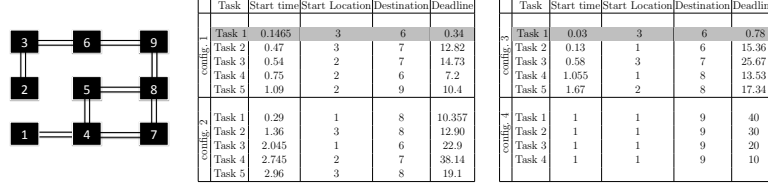


Fig. 1: The environment considered in our scenario and the four configurations used to verify our algorithm. Tasks marked with light-gray are those not accepted by the algorithm.

4 Results

We performed a set of experiments to validate and showcase the correctness of the proposed algorithm, to prove the utility of having a time planning phase, and to estimate the scalability performance for increasing robot density. We evaluate the algorithm performance in two planning strategies: *safe* planning and *risky* planning. With safe planning, the algorithm estimates the robot motion execution time (on a link h) as $\rho_i(h) = \mu + 3\sigma$ and thus the probability of missing the planned execution time is only 0.01. Instead with risky planning, the algorithm computes $\rho_i(h) = \mu + \sigma$ and thus the probability of missing the planned execution time of a link is 0.32. Testing these two strategies allows us to compare the predicted algorithm performances with our simulations' results. For simplicity, we assume that all links have the same length, and hence their execution times are sampled from a normal distribution with the same mean μ and standard deviation σ . The parameter α of Eq. (1) for computing the path weight is set to 0.5.

4.1 Validation case studies

We first start with verifying the correctness and efficacy of the proposed time-space planning algorithm for a multi-robot system of $N = 5$ identical robots through a set of agent-based simulation experiments. The environment that we consider in our experiments is depicted in Fig. 1, where robots can move between 9 partially connected locations—a small environment increases the chances of concurrent requests for a same link. In our simulations, we model the arrival of tasks using a homogeneous Poisson process with rate of 3 tasks/second. The tasks are generated with random start locations, destinations, start times, and deadlines. The start location is a node that is selected randomly between the nodes on the left side of the grid of Fig. 1 (i.e., either node 1, 2 or 3), while the destination is selected among nodes on the right plus node 6 (i.e., node 6, 7, 8 or 9). The task deadline D_i is randomly selected through a uniform distribution $\mathcal{U}(a_{i,0}, 3e_M)$ where e_M is the expected execution time of the longest path between the task's start location and its destination. We keep $\mu = 0.7$ in all experiments while we vary σ , with $\sigma = 0.1$ for safe planning and $\sigma = 0.3$ for risky planning. We let the algorithm to schedule paths for four task configurations, three of which were randomly generated (config 1, 2 and 3) and one manually chosen (config 4),

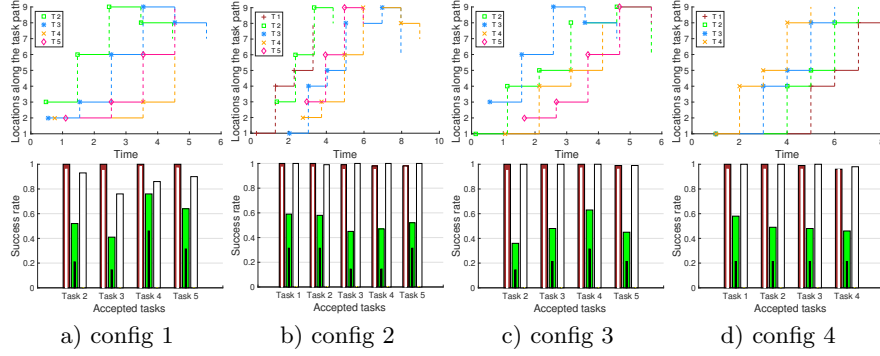


Fig. 2: Results of the agent-based simulations for the four considered task configurations. (upper part) Time-space plans generated by the algorithm (safe planning without re-planning). (lower part) Success rate for 100 simulation runs in three setups.

see the tables in Fig. 1. The plan is generated online while we simulate the task arrival and the robot motion through an agent-based simulator. We execute 100 runs for each configuration and for each planning strategy (i.e., safe and risky). The upper part of Fig. 2 shows the four plans generated by the algorithm (in safe planning with no re-planning). As previously defined, more tasks (i.e., robots) can stay simultaneously on the same node, however, a link can be used by at most a robot at a time. In the plots, solid (horizontal) lines represent a movement on a link departing from that node; the change of node is then visualised as a vertical dashed line of the same color. Preemption (pausing) of a task happens when the horizontal line is missing.

We can see that the algorithm generates plans that do not let two robots access the same link concurrently and produces plans where the accepted tasks meet their deadline. However, we can also see that in some cases (i.e., configs 1 and 3) the algorithm rejects two tasks (which have very short deadlines). Configuration 4 has been manually chosen and represents the case in which all tasks have the same start time, start location and destination while have different deadlines. In this example, we observe the sequential use of the links according to their respective deadlines. The lower part of Fig. 2 shows the success rate of 100 simulation runs, i.e., the proportion of runs in which the robot has reached its destination before its deadline. In each experiment, we simulate the motion of each robot through its planned path, which is generated online as soon as the new task request arrives. The robot motion time on each link is computed by drawing a random number from the probability distribution $\mathcal{N}(\mu, \sigma)$. When the robot motion is slower than planned and the robot misses a link deadline (see Eq. (6)), the algorithm needs to re-plan the task. We execute simulation experiments with three setups: (i) safe planning without re-planning if deadline are missed (red bars of Fig. 2(lower part)), (ii) risky planning without re-planning (green bars), and (iii) risky planning with online re-planning when robot motion has delays (white bars). The first two setups do not allow re-planning, therefore in case a

robot does not meet a link deadline, it stops and the task is considered as a failure. These two setups, without re-planning, allow us to match the predicted performances of the algorithm with the agent-based simulation results. In fact, the algorithm that operates in safe planning (i.e., using $\rho_i(h) = \mu + 3\sigma$ to estimate the robot motion execution time) predicts that its plans meets each link deadline more than 99% of the times. Therefore a path that is composed by k links is expected to have a success rate of 0.99^k . The simulation results match the predictions: the red bars of Fig. 2(lower part) are always above the respective white overlaying line which marks the lower bound of success. Similarly, the risky planning algorithm (which uses $\rho_i(h) = \mu + \sigma$) has 0.32 probability to fail on each link and, thus, a path composed of k links will succeed with probability greater than 0.68^k . Although we see a noticeable decrease in the success rate, the green bars of Fig. 2(lower part) are always above the predicted lower bound marks (black overlaying lines). The third set of simulations is performed to highlight the role of online re-planning. In this case, we allow the algorithm to re-plan the paths of the tasks which have missed a deadline on a link. While the algorithm operates with a risky planning strategy, we can appreciate that the system performance noticeably increases (compared to the no re-planning case, green bars).

4.2 Effect of time planning

To evaluate the utility of time-scheduling to access shared space, we compared our algorithm with a simple algorithm that assigns to each task its shortest path. Rather than applying time planning, it solves conflicts for accessing shared space resources choosing at random which path to divert (i.e., robots access shared links in arbitrary order). We use $\mu = 0.7$ and $\sigma = 0.3$ in all our experiments and we report results for varying number of task requests up to 40. As above, tasks are generated with random start locations, destinations, start times, and deadlines. For each data point, we generate 10 different sets of tasks and we simulate 30 task executions on each plan. The lines connect the average success rate and the vertical bar indicates the standard deviation of the 300 runs. Fig. 3a shows the proportion of completed tasks over the number of requested tasks. For being considered as completed, a task must be first accepted and scheduled by the planner, then the robot must execute the plan and reach the destination before the deadline. Instead, Fig. 3b shows the success rate which is computed as the rate of an accepted task to arrive at destination by its deadline.

The simple algorithm accepts all tasks, however many of them fails (i.e., have a very low rate in reaching destinations by their deadlines). Instead, the time-space planning refrains to accept a task that have a probability to miss its deadline above a certain threshold (which depends on the strategy whether it is risky or safe). For low number of tasks the performance of the three algorithms is comparable, however, when the number of tasks increases, our time-space solution largely outperforms the simple algorithm. Even if this result is expected because the simple algorithm does not have any strategy to deal with deadlines, the result displays the effectiveness and utility of the time planning phase.

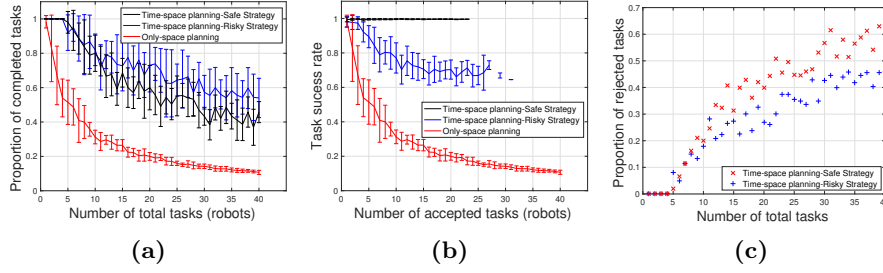


Fig. 3: Comparison between time-space planning (both with safe and risky strategy) and the simple algorithm without time planning phase, in terms of the (a) proportion of completed tasks and (b) task success rate. (c) Proportion of rejected tasks as a function of total number of requested tasks.

4.3 Scalability on number of tasks

Fig. 3a-c show how, respectively, the proportion of completed tasks, the success rate of a task and the proportion of rejected tasks changes as a function of the number of tasks. The plot of Fig. 3a shows that the risky strategy completes a higher number of tasks in average. This results is due to the fact that the risky planning accepts a larger number of tasks, however presents an higher risk of failure (with a success rate that converges to 0.68, in agreement with the $1\text{-}\sigma$ rule). Differently, the safe planning accepts fewer tasks but assures a higher success rate (around 0.99 according to the $3\text{-}\sigma$ rule). Additionally, we can see that while the simple algorithm accepts all tasks, the time-space algorithm does not accept anymore tasks above a certain upper bound, visualised through the interrupted lines (around 25 tasks) in Fig. 3b. This upper bound is determined by both the number of tasks (robot density) and the distance between deadlines. The idea behind time-space planning algorithms is to make a decision before the beginning of a task's execution about the probability of that task to succeed in meeting its deadline. A task begins only if it has a probability of success higher than a certain value, otherwise the task is rejected before beginning. This rejection mechanism has the goal to inform beforehand the user which may look for alternative solutions rather than missing the deadline halfway through.

5 Conclusions

We present an algorithm to plan paths for multiple robots that move in the same shared static environment. The algorithm plans new paths online as new requests arrive and manages possible conflicts if more robots want to access the same resource (space) at the same moment (time). The novel characteristic of the proposed time-space algorithm is its capability to plan paths that comply with temporal deadlines. Additionally, the algorithm is able to adapt online to unexpected delays in the robot motion which may be caused by internal robot failures/noise or by external factors that could hamper the normal movement. Finally, we show through agent-based simulations that the algorithm is able to

generate plans that respect the given deadlines with predictable performance levels. Natural extensions of this study consists in experimenting the algorithm performance in more challenging setups where each link has different congestion and the system is composed of heterogeneous robots, each with different motion speeds and reliability. Further extensions would consider dynamic environments (with time variant topologies), and larger link capacities.

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